EXERCISE – V   HINTS & SOLUTIONS

Sol.1 $y = 27^{\cos 2x} . 81^{\sin 2x}$

$= 3^{3\cos 2x} + 4\sin 2x$

$y_{min} = 3^{-5}$

$\cos(2x - \alpha) = -1$

$2x - \alpha = 2n\pi + \pi$

$x = (2n+1)\frac{\pi}{2} + \frac{\alpha}{2} \qquad n \in I$

& $y_{max} = 3^5$

$\cos(2x - \alpha) = 1$

$2x - \alpha = 2n\pi + 0$

$2x = 2n\pi + \alpha$

$x = n\pi + \frac{\alpha}{2} \qquad n \in I$

Sol.2 $5^{\mathrm{cosec}^2 x - 3\sec^2 y} = 1$

$\Rightarrow \mathrm{cosec}^2 x - 3\sec^2 y = 0 \Rightarrow \mathrm{cosec}^2 x = 3\sec^2 y$

$\Rightarrow \cos^2 y = 3\sin^2 x$ ....(i)

& $2^{2\,\mathrm{cosec}\,x + \sqrt{3}|\sec y|} = 64 = 2^6$

$\Rightarrow 2\mathrm{cosec}x + \sqrt{3}\ |\sec y| = 6$

$\Rightarrow \sqrt{3}\ |\sec y| = 6 - 2\ \mathrm{cosec}\ x$

$\Rightarrow 3\sec^2 y = 36 + 4\ \mathrm{cosec}^2 x - 24\ \mathrm{cosec} x$

$\Rightarrow \mathrm{cosec}^2 x = 36 + 4\ \mathrm{cosec}^2 x - 24\ \mathrm{cosec} x$

$\Rightarrow 3\mathrm{cosec}^2 x - 24\mathrm{cosec}\ x + 36 = 0$

$\Rightarrow (\mathrm{cosec} x - 6)(\mathrm{cosec} x - 2) = 0$

$\mathrm{cosec}\ x = 6$ or $\mathrm{cosec}\ x = 2$

$\Rightarrow \sin x = \frac{1}{6}$ or $\sin x = \frac{1}{2}$

$\sin x = \frac{1}{6}$ not satisfying second given eqn.

$2.6 + \sqrt{3}\ (+ve) \neq 6$

$\therefore \sin x = \frac{1}{2} \qquad \Rightarrow x = n\pi \pm (-1)^n \frac{\pi}{6}, n \in I$

$\cos^2 y = 3 \times \left(\frac{1}{2}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$y = n\pi \pm \frac{\pi}{6}, \ n \in I$

Sol.3 $7\cos x + 5\sin x = 2k + 1 \qquad k \in I$

$-\sqrt{7^2 + 5^2} \leq 2k + 1 \leq \sqrt{7^2 + 5^2}$

$-\sqrt{74} - 1 \leq 2k \leq \sqrt{74} - 1$

$- 9. ... \leq 2k \leq 7. ....$

$-4 .... \leq k \leq 3 ...$

$k = -4, 3, -2, -1, 0, 1, 2, 3$

No., of k is 8.

Sol.4 $\cos(\alpha - \beta) = 1 \quad \alpha, \beta \in [-\pi, \pi]$

$\Rightarrow \alpha - \beta = 0 \Rightarrow \alpha = \beta$

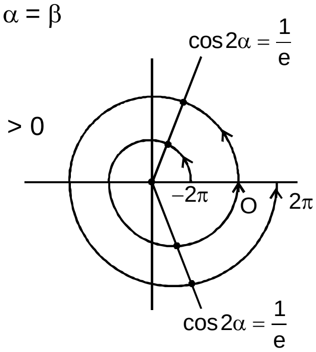
& $\cos(\alpha + \beta) = \frac{1}{e} > 0$

$\cos 2\alpha = \frac{1}{e}$

$-\pi \leq \alpha \leq \pi$

$-2\pi \leq 2\alpha \leq 2\pi$

no. of points of (α, β) are 4



Sol.5 $2\sin^2\theta - 5\sin\theta + 2 > 0$

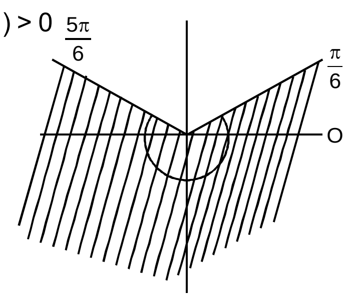
$\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$

$\because (\sin\theta - 2)(2\sin\theta - 1) > 0$

$\Rightarrow 2\sin\theta - 1 < 0$

$\Rightarrow \sin\theta < \frac{1}{2}$

$\theta = \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$



Sol.6 $2\sin^2\theta - \cos 2\theta = 0$

$1 - 2\cos 2\theta = 0$

$\cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$ .... (i) $\qquad n \in I$

$2\cos^2\theta - 3\sin\theta = 0$

$2 - 2\sin^2\theta - 3\sin\theta = 0$

$2\sin^2\theta + 3\sin\theta - 2 = 0$

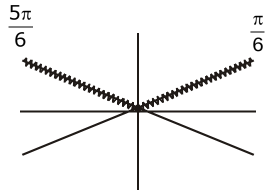
$\sin\theta + 2 \neq 0 \quad \therefore \quad \sin\theta = \frac{1}{2}$

$\theta = 2n\pi + \frac{\pi}{6}$ or $\theta = (2n + 1)\,\pi - \frac{\pi}{6}$ ....(ii)

from (i) of (ii)

common sol. are $\frac{\pi}{6}$ & $\frac{5\pi}{6}$

2 solution



Sol.7 $\sum_{m=1}^{6} \text{cosec}\left(\theta + (m-1)\frac{\pi}{4}\right)\text{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$

$\{ 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \sum \frac{1}{\sin\left(\theta + (m-1)\frac{\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$

$\Rightarrow \frac{1}{\sin\frac{\pi}{4}} \sum \left(\frac{\sin\left\{\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right\}}{\sin\left(\theta + (m-1)\frac{\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} \right) = 4\sqrt{2}$

$\Rightarrow \cot\theta - \cot\left(\theta + \frac{2\pi}{2}\right) = 4 \Rightarrow \cot\theta + \tan\theta = 4$

$\Rightarrow \tan\theta = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

$\theta = \frac{\pi}{12}, \theta = \frac{5\pi}{12}$

Sol.8 $\tan\theta = \cot 5\theta,\ \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\ \theta \neq \frac{n\pi}{5}$, for n= 0, ± 1, ± 2

$\Rightarrow \theta = n\pi + \frac{\pi}{2} - 5\theta,\ n \in I$

$\Rightarrow 6\theta = n\pi + \frac{\pi}{2} \Rightarrow 6\theta = (2n + 1)\,\frac{\pi}{2}$

$\Rightarrow \theta = (2n + 1)\,\frac{\pi}{12}$

$\therefore -\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} < (2n + 1)\,\frac{\pi}{12} < \frac{\pi}{2}$

$\Rightarrow -6 - 1 < 2n < 6 - 1 \quad n \in I$

$\Rightarrow -7 < 2n < 5 \Rightarrow -3.5 < n < 2.5$

$n = -3, -2, -1, 0, 1, 2$ ....(i)

& $\sin 2\theta = \cos 4\theta \Rightarrow \cos 4\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 4\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 6\theta = 2n\pi + \frac{\pi}{2}$ or $2\theta = 2n\pi - \frac{\pi}{2}$

$\Rightarrow \theta = (4n + 1)\frac{\pi}{12}$ or $\theta = (4n - 1)\frac{\pi}{4}$

$\Rightarrow -6 < 4n + 1 < 6$ or $-2 < 4n - 1 < 2$

$\Rightarrow -7 < 4n < 5$ or $-1 < 4n < 3$

$\Rightarrow -\frac{7}{4} < n < \frac{5}{4}$ or $-\frac{1}{4} < n < \frac{3}{4}$

$\Rightarrow n = -1, 0, 1$ or $n = 0$

$\Rightarrow \theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

common solution at n = –1, 0, 1

$\therefore \theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$ no. of sol. 3

Sol.9 $(y_0 + z_0)\cos 3\theta = x_0y_0z_0 \sin 3\theta$ ....(i) $y_0, z_0 \neq 0$

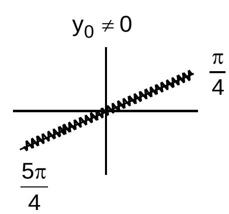
$x_0y_0z_0 \sin 3\theta = 2z_0\cos 3\theta + 2y_0 \sin 3\theta$ ....(ii)

$x_0y_0z_0 \sin 3\theta = (y_0 + 2z_0)\cos 3\theta + y_0\sin 3\theta$ ....(iii)

from (iii) – (ii) $y_0 \cos 3\theta = y_0 \sin 3\theta \quad y_0 \neq 0$

$3\theta = n\pi + \frac{\pi}{4}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$



From (i) & (ii)

$(y_0 - z_0)\cos 3\theta = 2y_0 \sin 3\theta$

& (i) (iii) $-2z_0 \cos 3\theta = 2y_0 \sin 3\theta$

$\Rightarrow (y + z) = 0$ & from (i) $x_0 = 0$

$\Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$ are satisfies all the given equation

$\therefore$ no. of values of θ is 3

Sol.10 D

for P $\Rightarrow \tan\theta = 1 + \sqrt{2} \Rightarrow \theta = n\pi + 3\pi/8$

for Q $\Rightarrow \tan\theta = \sqrt{2} + 1 \Rightarrow \theta = n\pi + 3\pi/8$

so p = Q